FIG. 6. Palatopterygoid and mandibular arches of a fourth individual from right side, with *hm*, hyomandibular.

FIG. 7. Superior tooth of external row, without apices of two of the cusps ; from the palatine bone of the specimen represented in fig. 5 ; one-half larger than nature, anterior view.

FIG. 8. Tooth of *Didymodus platypternus* Cope, nat. size, from above posteriorly.

FIG. 9. Tooth of a second specimen of *Didymodus platypternus* from below.

Photodynamic Notes, IX. By Pliny Earle Chase, LL.D.

(*Read before the American Philosophical Society, April 18, 1884.*)

411. *Æthereal Oscillation.*

Some readers of the Photodynamic Notes have found a difficulty in applying the laws of pendulum oscillation to the undulations of the luminiferous æther. It is well to guard against the conception of material pendulums, hung in or across the solar system, but it is also well to remember that the modern theories of molecular motion explain the rigidity of steel, and of all other solids, by the rapidity of motion, in ultimate discrete particles. If this view is correct, all changes in molecular movement are probably transmitted in and through the same elastic medium as the undulations of light, and all oscillations are in some way dependent on æthereal oscillations.

412. *Illustrations of Nodal Tendency.*

The well-known experiments of placing bits of paper on vibrating strings, sprinkling sand on Chladni plates, depositing fine powders in transparent musical tubes, and eliciting musical notes from glass vessels which are partly filled with water, illustrate the tendency of all vibrations to drive material particles towards musical nodes. These nodes are subject to the same laws of inertia which determine centres of oscillation in ordinary pendulums. The nodal tendency is greatest where the relative elasticity and the consequent undulatory velocity are greatest. As we know of no other medium in which the ratio of elasticity to density is so great as in the luminiferous æther, we can reasonably look in no other direction for such striking evidences of rhythmic influence as are to be found in cosmical and molecular arrangements.

413. *Æthereal Rotation.*

The supposed properties of the luminiferous æther are so similar, in many respects, to those of ordinary gases, that we may suppose it to act and react on all grosser forms and aggregations of matter. The rotations and revolutions of suns, planets and satellites are not only in harmony

with æthereal undulations, but they are also, as we may reasonably presume, produced by them. If cosmical rotation is dependent, in any way, upon æthereal waves, the reaction of cosmical inertia should produce a tendency to æthereal rotation.

414. *Extent of Rotating Influence.*

It is not unreasonable to suppose that the tendency to æthereal rotation at stellar centres, may be felt at a distance which is at least as great as the modulus of light. That distance in our system is very nearly equivalent to seventy-four times Neptune's mean radius vector. Although the rigidity in a rotating æthereal sphere may seem to be of a very different character from the rigidity of metallic rods and cosmical globes, it must evidently be accompanied by similar tendencies towards gravitating and oscillatory centres.

415. *Kinetic Postulates.*

All modern researches which have been guided by the theory of universal kinetic correlation seem to justify the following postulates:

1. An all-pervading, elastic æthereal medium, the particles of which are subject to gravitating attraction.
2. Consequent cyclic, rhythmic and harmonic tendencies of various kinds.
3. Probable frequency of simple forms of harmony, which are governed by centres of oscillation.
4. Mutual and equal action and reaction between centripetal gravitation and centrifugal radiation.
5. Radiating as well as projectile velocities, which are measured by the sum of cyclical resistances.
6. Correlation and mutual convertibility of light, heat, electricity, gravitation, etc.
7. Tendency of harmonic approximations to become numerically exact, as demonstrated by Laplace in discussing the motions and orbital periods of Jupiter's satellites.

416. *Importance of Reciprocals.*

In some text-books on arithmetic, a few lines are given to the explanation of reciprocals, and the statement is sometimes added, that the reciprocals of an arithmetical progression constitute a harmonic progression. Few, except those who devote themselves to a thorough scientific study of music, ever get any further knowledge of a subject which is full of interest, and which is likely to become of great importance in the future annals of scientific research. In his pamphlet on "Electrical Units of Measurement," Sir William Thomson enlarges upon the want of a unit of conductivity to represent "the reciprocal of the resistances." He says: "It is the conductivity that you want to measure, but the idea is too puzzling; and yet, for some cases, the conductivity system is immensely superior in accuracy and convenience to that by adding resistances in series."

417. *A Universal Want.*

Electricity is the form of force which is now, for many practical reasons, commanding general attention ; but its need of a more satisfactory and systematic study of reciprocal and harmonic activities is no greater than we can find in many other fields of physical research. Ohm's law brings all electrical phenomena so directly within the realm of resistance that Maxwell was inclined to regard electro-dynamics as more fundamental than thermo-dynamics. A full consideration of the subject would require a knowledge of mathematical principles which are somewhat intricate. There are many facts, however, which are so simple and intelligible that they may be easily learned, and a knowledge of them may awaken an interest which will facilitate investigation in every possible field.

418. *Spheral Music.*

We have all heard of the "music of the spheres ;" how many of us understand the literal truth of the statement :

> "There's not the smallest orb which thou behold'st,
> But in his motion like an angel sings."

The music of the spheres, as well as the music of the human voice, or of stringed or brazen instruments, is due to elasticity, which makes successive vibrations follow regular laws, so as to produce rhythmical and pleasing results. The beats of pendulums are governed by some of these harmonic laws and may be represented by harmonic formulas. The luminiferous æther, which is supposed to pervade all planetary and insterstellar spaces, and which Newton suggested as the possible storehouse of gravitation, should, on account of its enormous elasticity, furnish endless illustrations of faultless rhythm.

419. *Confirmation of the Hypothesis.*

We find, in accordance with the foregoing note, that the resistance of the sun to the interstellar vibrations of light produces a series of twenty-seven musical nodes, within the region in which solar attraction predominates over the attraction of the stars. Nine of the nodes are between Mercury and the Sun ; nine are at points which account for the positions of the eight primary planets and of the asteroidal belt ; and nine are between Neptune and the nearest of the fixed stars. The middle node of the middle nine, or the fourteenth node of the twenty-seven, is in the asteroidal belt. These facts, which have been already given in previous notes, are repeated in this connection as indicative of the probability that the æthereal rotation extends much further than was intimated in Note 414, and as giving the most stupendous evidence which has ever been published of the nodal tendencies to which reference is made in Note 411.

420. *Revelation.*

The foundation of all knowledge is revelation, which is always self-evident and infallible. The inspiration of the Almighty giveth understanding. All that we have and all that we are come from Him. In the interpretations of revelation, we are left in some measure to ourselves. While the self-evidence is given to us, we combine, in various ways, premises which we accept on account of their self-evidence or supposed self-evidence; in that combination we are liable to mistakes and fallacies of judgment. All truth is God's, all error is man's. They therefore make a fatal mistake who would set up the decisions of fallible judgment against the revelations which are offered for the acceptance of their own faith, or those which have been clearly apprehended through the faith of others, in truths which have been made self-evident to them.

421. *Fallacy of Agnosticism.*

We have no right to question the assertion of any individual that he does not know God. Neither has any one a right to say that God is unknowable. Receptivity, power, and knowledge, are the three fundamental axioms of all science and of all truth. So far as either of them is finite it is dependent upon something superior to itself. The agnostic, who recognizes a Supreme Power and who fails also to recognize a Supreme Receptivity and a Supreme Wisdom, has but a partial view. If in his teachings he implies, in any way, that human receptivity or human wisdom can be superior to any other receptivity or wisdom, he is guilty of arrogance and cannot shield himself under any assumption of humility. The only power of which we have any practical knowledge, is that of will; and will itself is always directed by purpose. So far as man, through the exercise of his purpose, his will and his intelligence, controls the powers of nature, he is imitating the Supreme control. Although it is true that we cannot "find out the Almighty unto perfection," and although it is also true that we should avoid any narrow anthropomorphism, there is no doubt that the purpose, the will, and the wisdom of man differ from those of the Almighty, not in kind, but only in degree, and that in these respects man has been created in the image of his Maker.

422. *The Oxygen Unit.*

Marignac (*Ann. de Chim. et de Phys.*, March, 1884), in his late re-examination of some of the atomic weights, considers that Prout's law is only approximate, and that, since the numbers which express the atomic weights only represent ratios, there is no reason for taking the hydrogen unit in preference to 16 or 100; but the choice of 16 is justified by its practical advantage. It allows us to represent the atomic weights of the greatest number of elements, and especially of those which are most important, by the most simple possible integers and with the least difference from the rigorous results of experiment. The fact that the atomic weights exhibit

more exact ratios to the oxygen than to the hydrogen unit, appears to have been first pointed out in No. 138 of the foregoing notes.

423. *Universal Rotation.*

The hypothesis that every material particle is endowed with rotation, by which it represents a definite amount of living force, has often been broached. Its probability is strengthened by the magnetic theories* of Arago, Ampére, Barlow, Lecount, Challis, Babbage, Herschel, Christie, Maxwell, Imray, Forbes and others; by my own investigations confirmatory of the hypothesis that "there can be no weight without some degree of momentum;"† by the connection of magnetism with rotation in a magnetic field through Laplace's principle of periodicity (note 333); and by the evidences which are furnished, by notes 413 and 419, of interstellar æthereal rotation, producing nodes which are determined by stellar moduli of light. According to this hypothesis no material particle can be wholly divested of energy, and no particle can ever acquire energy enough to free it from the equilibrating tendencies which spring from the law of equal action and reaction.

424. *Nascent Nebular Rotation.*

The beginning of the transfer of rotation from æthereal particles to cosmical masses, is illustrated by the equivalence of ratios between masses and rupturing distances, in the two ruling globes of the solar system. Taking Bessel's estimate of the mass-ratio of Sun to Jupiter (1047.879), the vector-radii of the two bodies, when in static equilibrium with regard to their common centre of gravity, should be in the same ratio. The projectile energy, which changed the static into an oscillatory dynamic equilibrium, has produced a secular eccentricity, according to Stockwell, of .0608274, the secular perihelion being, therefore, .9391726 of Jupiter's mean radius vector. Dividing the static ratio of vector radii, 1047.879, by .9391726, we get 1115.7469×Sun's semi-diameter for Jupiter's mean radius vector. Dividing this value by 5.202798, we get 214.4513 for Earth's mean radius vector, which represents a mean solar apparent semi-diameter of 961″.8254. The British Nautical Almanac estimate is 961″.83.

425. *Nascent Resistance.*

Laplace's principle of periodicity, and incipient "subsidence," according to Herschel's modification of the nebular hypothesis, are both exemplified in the equation:

$$(1 - e)\, g_5 t_5 = \sqrt{g_0 r_0}$$

in which e = Jupiter's secular eccentricity; g_0, g_5 = gravitating acceleration at the equatorial surface of Sun and Jupiter, respectively; t_5 = time

*Cited in Proc. Amer. Phil. Soc., ix, 356-8, 367-9, 491; Proc. Roy. Soc., xxii, 352-3, etc.

†Proc. Amer. Phil. Soc., ix, 357, 492.

of Jupiter's half rotation = 17863.25 sec. ; r_0 = Sun's semi-diameter.
Hence $\sqrt{g_0 r_0} = 2\pi \times 214.4513^{\frac{3}{2}} r_0 \div 31558149 = .0006252614\ r_0$.

$$g_0 = .0000003909518\ r_0$$
$$g_5 = .00000003727\ r_0 = .09533064\ g_0$$
$$r_5 = .10005233\ r_0$$

426. *Nascent Centre of Condensation.*

The incipient subsidence of Jupiter, as indicated by the factor $(1 - e)\ g_5$, coöperates with solar attraction in the formation of a belt of maximum condensation. Accordingly, the second planetary mass, in regard to the simplicity of harmonic relations, is Earth, which occupies the centre of the dense belt. Its distance from the Sun and its mass may be found by means of the equation

$$\sqrt{(1 - e)\ g_5 r_5} = (\rho_5 \div \rho_3)\ \sqrt{g_3 r_3}$$

The mean radius vector is designated by ρ, Jupiter and Earth being indicated by subscript $_5$ and $_3$ respectively. We have, therefore,

$$g_3 = .00607723 \text{ miles}$$
$$r_3 = 3962.8 \text{ miles}$$
$$\sqrt{g_3 r_3} = 4.90743 \text{ miles}$$
$$\rho_5 \div \rho_3 = 5.202798$$
$$\sqrt{(1 - e)\ g_5 r_5} = .09464615\ \sqrt{g_0 r_0} = .0000591786\ r_0$$
$$r_0 = 431445.64 \text{ miles}$$
$$\rho_3 = 214.4513\ r_0 = 92524100 \text{ miles.}$$
$$\sqrt{g_0 r_0} = 269.766 \text{ miles}$$
$$m_0 \div m_3 = 328997$$
$$r_5 = 10.893\ r_3$$

427. *Nascent Nodal Harmonies.*

The formation of a belt of maximum condensation, by the action and reaction of subsidence and rotation between the two principal masses of the system, establishes the conditions which are requisite for nodal harmonies of various kinds. One of the simplest harmonic series is $\frac{2}{1}$, $\frac{2}{3}$, $\frac{2}{5}$, etc. The centre of linear oscillation adds its influence to the natural rhythmic tendencies of the second of these nodes. Both the moment of rotary inertia of a thin spherical shell and the nodes of aggregating collision in condensing nebulæ* also introduce the factor $\frac{2}{3}$, and the moment of a rotating æthereal or other homogeneous sphere introduces the factor $\frac{2}{5}$. Moments of inertia vary as distances of projection against uniform resistance ; we may, accordingly, look for the frequent recurrence of the factors $\frac{2}{3}$ and $\frac{2}{5}$, in the harmonic rupturing nodes of condensing and rotating nebulæ, especially in the neighborhood of the most important centres of condensation.

*Proc. Am. Phil. Soc., xvii, 99.

428. *The Sun-Earth Balance.*

The situation of Earth's orbit, between the orbit of Jupiter and the Sun, introduces tendencies to condensation and rotation of the character referred to in the foregoing note. The action and reaction of æthereal waves, between the principal centres of attraction and of condensation, have produced an amount of gravitation, at the earth's equatorial surface, which is sufficient to give a circular orbital velocity of $\sqrt{gr} = 4.90743$ miles per second. The linear oscillation of the Earth around the Sun, as well as the centre of rotary inertia for a superficial film of condensation or of luminous undulation in the orbit of Mars, multiplies this energy by $\frac{3}{2}$; the rotary æthereal oscillation of a sphere which has its limit in the asteroidal belt also multiplies the energy by $\frac{5}{2}$. Accordingly, if the Earth's orbit was always circular, its velocity of revolution, as thus determined, would be $\frac{5}{2} \times \frac{3}{2} \times 4.90743 = 18.40286$ miles per second. There are 31558149 seconds in a year, therefore the Sun's distance should be, if Earth's orbit were always circular, $18.40286 \times 31558149 \div 2\pi = 92430800$ miles. This is probably correct within less than $\frac{1}{8}$ of one per cent. (See Note 434.)

429. *Accuracy of Harmonic Method.*

The above method of estimating the Sun's distance is the shortest which has ever been published. I believe that it is also the most accurate if proper allowance is made for orbital eccentricity, for the following reasons :

1. If the hypothesis of an all pervading luminiferous æther is true, all its cyclical movements must be rhythmic, or harmonic, the various forms of rhythm being governed by various centres of oscillation.

2. The simplest kinds of oscillatory motion, in cosmical bodies, are linear and spherical.

3. Laplace showed, in discussing the motions of Jupiter's satellites, that whenever there are tendencies to simple numerical relations, in planetary arrangements, all the forces of the system combine to make those tendencies exact.

4. The Sun is the principal centre of attraction, and the Earth is the principal centre of condensation in the solar system.

430. *Rhythmic Weight of the Sun.*

The Sun can be weighed by its musical rhythm with a corresponding facility. Orbital velocities vary inversely as the square root of the distance from the centre of gravity. Any two attracting bodies bear the same ratio to each other as the distances at which they would communicate equal orbital velocities, to particles which revolve about the centre of gravity of the attracting bodies. Hence we have :

Earth's Radius Vector. Earth's Radius.

$$92430800 \times (\tfrac{5}{2} \times \tfrac{3}{2})^2 : 3962.8 : : 328002 : 1.$$

In other words the Sun would weigh 328002 times as much as the Earth, if Earth's orbit were always circular.* The remarkable accordances among the various harmonic estimates which are deduced from the correlations of mechanical, electrical, chemical, luminous and other forces, indicate an amount of probable error which is much smaller than those of ordinary astronomical estimates.

431. *Lunar Mass. First Estimate.*

Ferrel (*Methods and Results*, p. 20) gives 7989 metres as the height of the homogeneous atmosphere. The equilibrium of atmospheric elasticity, between the mutual interactions of Earth and Moon (Notes 8, 316), gives the following proportion :

$$(20000000 \div \pi) : 7989 : : r_3 : .0012549 r_3 : : (\pi^2 \times 80.74) : 1$$

432. *Lunar Mass. Second Estimate.*

The estimates of the height of a homogeneous atmosphere differ for different latitudes and for slight variations in the elements of the calculation. It may, therefore, be more satisfactory to deduce the Moon's mass from the simple principles of oscillation.

From Notes 8, 162, 246, we find :

$$l = g\left(\frac{t}{\pi}\right)^2 = \frac{32.08776}{5280} \times (43082.04)^2 \div \pi^2 = 1142874 \text{ miles}$$

for the length of Earth's theoretical pendulum. From this equation we deduce the ratio of Earth's mass (m_3) to Moon's mass (μ), by the proportion :

$$\rho_3 : l : : m_3 : \mu.$$
$$92524100 : 1142874 : : 80.957 : 1$$

433. *Earth's Secular Eccentricity.*

The harmonic relations of the Earth and Moon are still further shown by the evidences of original terrestrial projection before the Moon separated from the Earth. If we designate Earth's secular perihelion radius vector by ρ'_3, we have the proportion, $g_o r_o (m_3 + \mu) : (g_3 t_3)^2 m_3 : : \rho_3 : \rho'_3$.

In other words, the orbital vis viva of original solar projection, for the combined masses of Earth and Moon, is represented by the mean radius vector, while the limiting oscillatory vis viva of the Earth alone is represented by the radius vector of secular perihelion. Substituting in the above proportion the harmonic values which we have already found, we have

$$269.766^2 \times 81.957 : 261.8194^2 \times 80.957 : : 1 : .930462$$

this gives, for Earth's secular eccentricity, .069538.

Stockwell's estimate of this eccentricity for the value of Earth's mass which we have deduced from its harmonic oscillation is .06901. The dif-

*See Note 434.

ference between his perihelion radius vector and the corresponding harmonic radius is less than $\frac{1}{17}$ of one per cent.

434. *Correction for Secular Eccentricity.*

In Note 428, Sun's distance was estimated upon the hypothesis that Earth's orbit was circular. The mean distance, however, may be considered as having been established at the time of original rupturing projection, or, in other words, at secular perihelion. The circumference of an ellipse is $2\pi a\left(1-\frac{1}{2^2}e^2-\frac{1^2.3}{2^2.4^2}e^4-\frac{1^2.3^2.5}{2^2.4^2.6^2}e^6, \text{ etc.}\right)$

Substituting the theoretical value $e = .06954$, this becomes $2\pi a \times 99879$. The corresponding value of Earth's mean radius vector is $92430800 \div .99879 = 92542790$, which differs by less than $\frac{1}{49}$ of one per cent, from the value which was deduced in Note 426, from the incipient subsidence of Jupiter. The corresponding value of $m_0 \div m_3$ is 329196.

435. *Twin Planets.*

Action and reaction, in a system which is fundamentally dependent upon two largely preponderating bodies, may naturally lead to a grouping in pairs. Laplace's modification of the nebular hypothesis, which supposes that the first ruptures are in the form of rings or belts, and Herschel's hypothesis of subsidence until the acquired velocity becomes rupturing, also favor the simultaneous formation of companion perihelion and aphelion planets. Accordingly, we find two supra-asteroidal groups, Neptune-Uranus, Jupiter-Saturn, and two infra-asteroidal, Mars-Mercury, Earth-Venus. The grouping in the belt of greatest condensation indicates a double tendency; Earth-Venus representing influences which appear to have orignated in the Sun, while Mars-Mercury seem to be more specially referable to activities at the centre of condensation, than to those at the centre of nucleation.

436. *Mass Relation of Jupiter and Saturn.*

The discovery, by Prof. Stephen Alexander, that the masses of Jupiter and Saturn are nearly in the inverse ratio of the squares of their mean vector-radii, was the first step towards a demonstration of the fundamental principles of harmonic astronomy. This ratio represents the moments of æthereal or nebular rotary inertia for the two planets, respectively. The closeness of the approximation is shown by the proportion

$$5.202798^2 : 9.53852^2 :: 104.879 : 3522.3.$$

Hall's estimate of Sun $\div$ by Saturn is 3482; Bessel's 3501.6; Leverrier's 3512. The greatest difference between either of these estimates and Alexander's approximation is only about $1\frac{1}{6}$ per cent, the least difference is less than $\frac{1}{8}$ of one per cent. I know of no other mass-approximations which rest upon purely rhythmical laws, except my own.

437. *Mass-Relation of Earth and Venus.*

The simplicity of the harmonic mass-relations between Sun and Earth, as well as between Jupiter and Saturn, increases the likelihood of similar relations, which are equally simple, between Earth and Venus; but the wide range of discrepancy among the estimates of different astronomers makes it somewhat difficult to ascertain what rhythmic influence has prevailed. Stockwell's estimate of Venus's mass is about .945 of Earth's mass; Hill's is only .831; Leverrier's mean of two estimates, .872. The value which would give Earth and Venus equal orbital momenta is .85049. We may, therefore, claim a great probability for the proportion, $m_0 : m_3 :: (329196 \div .85049 = 387066) : 1$.

438. *Mass-Relation of Neptune and Uranus.*

In the exterior twin planet belt, we find a harmonic mass relation which is no less striking than those that have already been given. It is especially interesting, as pointing to an early stage of nebular condensation, as well as to a blending of external and internal influences which accounts for retrograde satellite rotation and revolution. The gravitating accelerations, which are due to the actions and reactions between two cosmical masses, are proportioned to the respective masses. The vis viva of gravitating subsidence $\left(\frac{m\,v^2}{2}\right)$ is, therefore, proportioned to the cube of the masses. We find, accordingly, that

$$m_8^3 : m_7^3 :: \rho_8' : \rho_7.$$

In this proportion ρ_8' represents the locus of incipient subsidence, or secular aphelion of Neptune, while ρ_7 represents the mean radius vector of Uranus. The values which satisfy this proportion are very exact, as will be seen by the following comparison:

	Harmonic.	Newcomb.
Sun ÷ Neptune	19372.86	19380 ± 70
Sun ÷ Uranus	22603.33	22600 ± 100

439. *Mass-Relation of Mars and Mercury.*

The Earth appears to have exercised an influence upon the two exterior planets of the belt of greatest condensation, analogous to that which the Sun has exercised upon Neptune and Uranus. We find, accordingly,

$$m_4^3 : m_1^3 :: \rho_4 : \rho_1.$$

In this proportion, if we let ρ_4 and ρ_1 represent the mean distances of Mars and Mercury, respectively, the mass of Mars would be 1.5789 times that of Mercury. Adopting Hall's estimate of Sun ÷ Mars = 3093500, we find Sun ÷ Mercury = 4884366. Encke s estimate is 4865751. These estimates are based upon the hypothesis that Sun ÷ Earth = 354936. If we substitute the harmonic value, 329196, we get Sun ÷ Mars = 2869151; Sun ÷ Mercury = 4530150.

440. *Linkage of Earth and Neptune.*

It was shown in Note 428 that Earth's mass may be harmonically deduced from Sun's mass through its limiting value of circular orbital velocity. Neptune's mass may be deduced from Earth's through the corresponding limit of orbital time. We have, accordingly, $m_8 : m_3 : : t_a : t_\beta : :$ 16.98 : 1. In this proportion $t_a = 2\pi \sqrt{\frac{r}{g}}$; t_β = a sidereal day.

441. *Earth's Oblateness.*

The importance of Earth's position, at the centre of the belt of greatest condensation, is further shown by the fact that its centrifugal force of daily rotation, by which it is harmonically connected with Neptune, has also determined its oblateness. For we find that $(t_a \div t_\beta)^2 = 288.4$. Listing's estimate (See Note 249) is 288.5.

442. *Linkage of Earth and Uranus.*

Another interesting connection between the dense belt and the outer twin-planet belt, is shown by the proportion, 365.2565 : 338.2183 : : ρ_6' : ρ_6. In this proportion, 338.2183 is the distance, measured in Earth's semidiameters, at which a satellite particle would revolve in a solar year; ρ_6' is the secular aphelion distance of Uranus, while ρ_6 is its mean distance. This relation is also interesting because the aphelion of the 33.25 year meteoric belt is in the orbit of Uranus, and because a ray of light would traverse the same meteoric orbit in the time of one solar rotation. The proportion gives, for the secular aphelion of Uranus, 1.07994. Stockwell's estimate is 1.07797.

443. *Another Linkage of Earth and Jupiter.*

In notes 425 and 426, the gravitating accelerations of Earth and Jupiter were shown to be harmonically related to each other, as well as to the gravitating acceleration of the Sun. The moon furnishes another harmonic link, which is shown by the equation

$$60.2778 \times 5.202798 \times 1047.879 = 328629.$$

In this equation 60.2778 is von Littrow's estimate of Moon's mean distance in equatorial semidiameters of the Earth; 5.202798 is Jupiter's mean distance, in Earth's mean vector radii; 1047.879 is the quotient of Sun's mass by Jupiter's mass; 328629 is, within less than $\frac{1}{5}$ of one per cent, the harmonic quotient of Sun's mass by Earth's mass.

444. *The Meteoric Theory of World Building.*

Proctor (*North American Review*, May, 1884) criticises the theory of Olbers, which has been lately advocated by Herbert Spencer, and considers that the asteroidal belt has been formed by meteoric influence, in connection with the attraction of Jupiter, rather than by the explosion of any primitive planet. In many of the foregoing notes there has been evidence

of influences which may be regarded as meteoric. Indeed, Herschel's subsidence-theory recognizes the continual activity of such influences, provided we consider every particle which is falling towards the sun as meteoric. If we still further regard the luminiferous æther as material, we may consider ourselves as living in a condensing and rotating nebula.

445. *Linkage of Sun, Earth, Jupiter and Saturn.*

The influence of simple primitive subsidence, which was so strikingly exemplified in the mass-ratios of Neptune and Uranus (Note 438), is no less evident in the four important cosmical bodies which represent, respectively, the chief centre of nucleation (Sun), the chief centre of condensation (Earth), the primitive nebular centre (Jupiter), and the centre of inertia of the primitive planetary system (Saturn). This influence is shown by the equation

$$\text{Sun} \times \text{Earth} \times \text{Saturn} = \text{Jupiter}^3.$$

Substituting in this equation the harmonic ratio of Sun to Earth (329196) and Bessel's estimate of Sun ÷ Saturn (3501.6), we get for Sun ÷ Jupiter 1048.5.

446. *Saturn's Secular Eccentricity.*

The mutual actions and reactions, among the four cosmical masses which were introduced into the foregoing note, are still further shown by the connection of the orbital periods of Earth and Jupiter with the secular eccentricity of Saturn.

This connection is shown by the proportion

$$4332.5848 : 365.2565 : : 1 : .0843045.$$

Stockwell's estimate of Saturn's secular eccentricity is .0843289.

This differs by less than $\frac{1}{3\,4}$ of one per cent from the harmonic estimate.

447. *Primitive Phyllotactic Relations.*

The centre of a nebula which is bounded at opposite extremities of its diameter by the secular aphelia, or loci of incipient subsidence, of Neptune and Uranus, according to Stockwell's estimate, is 4.8952. This differs by less than $\frac{1}{5}$ of one per cent from Jupiter's secular perihelion, or locus of incipient nebular rupture. The mass of Neptune is approximately $\frac{3}{2}^{\frac{1}{4}}$ of Earth's harmonic mass. Uranus is almost precisely $\frac{6}{1\,3}$ of the Uranus-Neptune belt. Saturn is almost precisely $\frac{3}{1\,3}$ of the Jupiter-Saturn belt. The numbers $\frac{1}{2}$, $2 \times \frac{3}{1\,3}$, $\frac{2}{3\,4}$ and $\frac{3}{1\,3}$ are all phyllotactic. The values which fully satisfy these mass relations are

Sun ÷ Neptune	19352.76
Sun ÷ Uranus	22578.22
Sun ÷ Saturn	3490.71
Sun ÷ Jupiter.	1047.21

448. *Phyllotaxy in the Asteroidal Belt.*

Kirkwood (Proc. Amer. Phil. Soc., xxi, 266) in discussing the gaps and clusters of the Asteroidal belt, says: "In three portions of the ring the clustering tendency is distinctly evident. These are from 2.35 to 2.46, from 2.55 to 2.80, and from 3.05 to 3.22; containing forty-three, ninety-six and forty asteroids, respectively. We have thus an obvious resemblance to the rings of Saturn; the partial breaks or chasms in the zone corresponding to the well known intervals in the system of secondary rings." He accounts for the gaps by the periodic harmonic perturbations of Jupiter, but he gives no explanation of the clustering tendency.

If we take $\frac{6}{13}$, $\frac{1}{2}$, and $\frac{5}{8}$ of Jupiter's mean distance, we have 2.401, 2.601, 3.252. The numbers $2 \times \frac{3}{13}$, $\frac{1}{2}$, and $\frac{5}{8}$ are all phyllotactic. The first of the clusters, $\frac{6}{13}$, seems to indicate a harmonic connection with the primitive rupture of the Uranus-Neptune belt which was pointed out in the foregoing note.

449. *Constant of Aberration.*

Magnus Nyrén has published, in the Memoirs of the St. Petersburg Academy, a valuable paper on the determination of the constant of aberration. A summary of his results is given by A. M. W. Downing, in *The Observatory*, vi, 365. The value which has long been accepted by astronomers is 20″.445. Struve discussed the possible sources of error, some years after the publication of his memoir, and adopted the value 20″.463. Nyrén deduces, from three different sets of observations at Pulkowa, $20''.492 \pm 0''.006$, which Downing thinks "must be an extremely accurate value of this important constant, and will probably have to be considered final until it can be corrected by an equally accurate and extensive series of determinations made in the southern hemisphere. Such a determination is, at the present time, a desideratum in astronomy."

450. *Succession of Harmonic Mass Influences.*

According to the foregoing notes, the first harmonic influence in the determination of relative planetary masses seems to have been that of simple subsidence, represented by the cubes of masses. Next was the simple product of mass by distance, representing the beginning of the change from static to rotary equilibrium. Then came the product of mass by the square of the distance, representing nebular rotary inertia. This was followed by the quotient of mass by the square root of distance, representing simple orbital momentum. These relations seem so natural and so important that it may be well to give the calculations in detail, for future reference, and also to extend those calculations to the most important linkages which have been indicated among the different planetary and satellite belts.

451. *Simple Subsidence.*

There are three planetary illustrations of the determination of mass by simple subsidence:

1. In the Neptune-Uranus belt (Note 438) the mass of the belt (m_7 +

m_8) and the mean eccentricity of the outer or subsident member are so influenced by the change of centripetal into tangential orbital motion that we find

$$\pi^2 (1 + e_8) (m_7 + m_8) = m_5.$$

According to Stockwell, Neptune's mean secular eccentricity, $e_8 =$ .0100389. Hence we derive the data for the following calculation:

	30.46955	1.4838661	1
	19.183581	1.2829297	2
$\frac{1}{3}$ (1 — 2)	1.1667527	.0669788	3
π^2		.9942998	4
$1 + e_8$		.0043381	5
		3.0203111	6
(4 + 5 + 6) = log. $1 \div (m_7 + m_8)$		4.0189490	7

log. (3) representing the ratio of $m_8 : m_7$, log. $m_8 =$ log. $\frac{1.1667527}{2.1667527}$ + (7), and log. $m_7 =$ log. $\frac{1}{2.1667527}$ + (7),

2.1667527		.3358093	8
(7 + 8) = $1 \div m_7$		4.3547583	9
(9 — 3) = $1 \div m_8$		4.2877795	10

2. In the actions and reactions of the chief centres of nucleation (Sun), condensation (Earth), nebulosity (Jupiter), and planetary inertia (Saturn), the mass relation arises which is given in Note 445.

$1 \div m_3 = 329196$		5.5174544	11
$1 \div m_6 =$ 3501.6		3.5442665	12
$\frac{1}{3}$ (11 + 12)		3.0205736	13

3. In the Mars-Mercury belt, as modified by solar and terrestrial action (Note 439).

$\rho_4 =$	1.5236898	.1828960	14
$\rho_1 =$	.3870987	$\bar{1}$.5878218	15
(14 — 15)		.5950742	16
$\frac{1}{3}$ (16)	1.5789	.1983581	17
	3093500	6.4904501	18
(17 + 18)	4884366	6.6888081	19
	354936	5.5501499	20
(18 + 11 — 20)	2869151	6.4577546	21
(19 + 11 — 20)	4530150	6.6561126	22

452. *Change of Static to Rotary Equilibrium.*

The following logarithms represent the influence of the change from static to rotary equilibrium as explained in Note 424.

$1 \div m_5 =$	1047.879	3.0203111	23
ρ'_5	.9391726	$\bar{1}$.9727454	24
(23 — 24)	1115.747	3.0475657	25
ρ_5	5.202798	.7162369	26
ρ_3	214.4513	2.3313288	27
$r =$	206264″.806247	5.3144251	28
(28 — 27) $r_0 =$	961″.8254	2.9830963	29

453. *Mass of Saturn.*

Dr. Meyer's estimate of $m_0 \div m_6$ (*The Observatory*, vi, 279), is 3482.93 $\pm$ 5.5. This is nearly identical with Hall's value, as given in Note 436. Meyer includes the rings, estimating their mass as equivalent to $\frac{1}{119.1} m_6$. Bessel's estimate was $\frac{1}{118}$. If we omit the rings, $m_0 \div m_6$ becomes, according to Meyer, 3512.2, which is substantially identical with Leverrier's value (3512). Substituting in Alexander's harmony we get, for Saturn's mean distance, $9.52513_{\rho 3}$, which differs by less than $\frac{1}{7}$ of one per cent from the generally accepted distance.

	3512.2	3.5455792	30
(26) + ½ (30 — 23)	9.52513	.9788709	31

454. *Orbital Momentum.*

The logarithms which represent orbital momentum for Earth and Venus (Note 437) are as follows :

	.7233323	$\bar{1}$.8593379	32
½ (32)	.85049	$\bar{1}$.9296690	33
(11 — 33)	387066	5.5877854	34

Notes 451–4 give simple harmonic approximations to all the masses of the eight primary planets. I think this is the first publication in which a uniform scale has been adopted for all the planets, and I submit it fearlessly for comparison with any estimate which has been based on ordinary astronomical data.

455. *Linkage of Earth and Jupiter.*

We are now prepared for a systematic examination of the linkages among the different belts. In note 425 we found that the gravitating energies, at the chief centres of nucleation and of nebulosity, are so connected as to give the equation :

$$(1 - e)\, g_5\, t_5 = \sqrt{g_0\, r_0}.$$

The value of $\sqrt{g_0\, r_0}$ may be found by the following logarithms :

2π		.7981799	35
⅔ (27)		3.4969932	36
31558149		7.4991115	37
(35) + (36) — (37)		$\bar{4}$.7960616	38
2 (38)		$\bar{7}$.5921232	39
$t_5 =$	17863.25	4.2519605	40
(38) — (24) — (40)		$\bar{8}$.5713557	41
(41) — (39)		$\bar{2}$.9792325	42
½ (— 23 — 42)		$\bar{1}$.0002282	43

456. *Earth's Mean Radius Vector.*

The mean radius vector of the chief centre of condensation is harmonically found by the methods of Notes 428 and 434.

$g_3 = .00607723$ miles.	$\overline{3}.7837055$	44
$r_3 = 3962.8$	3.5980022	45
$\sqrt{g_3 r_3} = 4.90743$	.6908538	46
$\frac{5}{2} \times \frac{3}{2}$	.5740313	47
(46) + (47)	1.2648851	48
(48) + (37) — (35)	7.9658167	49
99879	$\overline{1}.9994742$	50
(49) — (50)	7.9663425	51

The value of $m_0 \div m_3$, which represents this mean distance, may be found by the formula:

$$4\left(\frac{\rho_3}{r_3}\right)^3 \pi^2 r_3 \div (g_3 \times 31558149^2) = m_0 \div m_3$$

$$3\,(51 - 45) + 2\,(35) + (45) - (44) - 2\,(37) = (11)$$

457. *Second Linkage of Earth and Jupiter.*

The linkage between the superficial gravitating energies of Earth and Jupiter, which is shown in Note 426, may be computed as follows:

$\frac{1}{2}$ (24 + 42 + 43)	$\overline{2}.9761030$	52
(38 + 52)	$\overline{5}.7721646$	53
(26 + 46 — 53)	5.6349261	54
(27 + 54)	7.9662549	55
(38 + 54)	2.4309877	56
2 (56 — 46) + (54 — 45)	5.5171917	57
(43 + 54 — 45)	1.0371521	58

458. *Other Terrestrial Linkages.*

The linkage of Earth, Moon and Jupiter, which is represented in Note 443, introduces the following logarithms.

60.2778	1.7801574	59
(59 + 26 + 23)	5.5167054	60

The linkage of Earth and Neptune (Note 440) with Earth's oblateness gives the following logarithms:

$\frac{1}{2}$ (45 — 44)		2.9071484	61
(61 + 35)		3.7053283	62
86164.08		4.9353263	63
(63 — 62)	16.9824	1.2299980	64
(11 — 64)	19384.58	4.2874564	65
2 (64)	288.4	2.4599960	66

The linkage of Earth and Uranus, Note 442, gives the following logarithms :

365.2565		2.5625979	67
338.2183		2.5291971	68
(67 — 68)	1.07994	.0334008	69

459. *Moon's Mass and Earth's Eccentricity.*

The harmonies of lunar mass and Earth's orbital eccentricity (Notes 431–3) introduce the following logarithms :

π		.4971499	70
7989		3.9024924	71
20000000		7.3010300	72
(70 + 71 — 72)	.0012549	$\bar{3}$.0986123	73
— (73)		2.9013877	74
(74) — 2 (70)	80.74	1.9070879	75
43082.04 = t_3		4.6342963	76
(44) + 2 (76) — 2 (70)		6.0579983	77
(55 — 77)	80.957	1.9082566	78
81.957		1.9135861	79
(44 + 76)		2.4180018	80
2 (80) + (78) — 2 (56) — (79)		$\bar{1}$.9686987	81

460. *Series of Harmonic Equations.*

The harmonic analogy between the Neptune-Uranus and the Mars-Mercury belts may be still further extended by the following equations, which enable us to deduce all the masses of the primary planets from the harmonic value of the mass at the chief centre of condensation :

$g = \frac{v_\lambda\, m}{t\, r^2}$	α
$\frac{3}{2} \times \frac{5}{2} \times \sqrt{g_3\, r_3} = \sqrt{\gamma_3\, \rho_3}$	β
$t_\alpha \div t_\beta = m_8 \div m_3$	γ
$\pi^2\,(1 + e_8)\,(m_8 + m_7) = m_5$	δ
$\pi^2\,(m_1 + m_4) = m_2 + m_3$	ε
$\rho_8' \div \rho_7 = m_8^3 \div m_7^3$	ζ
$m_0\, m_3\, m_6 = m_5^3$	η
$\rho_1 \div \rho_4 = m_1^3 \div m_4^3$	θ
$\rho_2 \div \rho_3 = m_2^2 \div m_3^2$	ι

461. *Explanation.*

In the foregoing note g = gravitating acceleration of any mass m, at any distance r, provided m and r are expressed in units of Sun's mass and semi-diameter ; v_λ = velocity of light ; t = time of solar half-rotation ; γ_3 = solar gravitating acceleration at Earth's mean radius vector (ρ_3) ; t_β = time of theoretical satellite rotation at Earth's equatorial surface =

$2\pi\sqrt{\frac{r_3}{g_3}}$; t_a = a sidereal day; e_8 = Neptune's minimum secular eccentricity; ρ_8' = Neptune's mean secular aphelion; ρ_1 to ρ_8 = mean vector radii and m_1 to m_8 = masses of the eight primary planets.

The equations represent various obvious radial and tangential actions and reactions. Equation (1), when applied at the Sun's surface, which is the point of greatest gravitating acceleration in the solar system, gives $g\,t = v_\lambda$. This satisfies Ohm's law, as applied to solar rotation in a magnetic field, Fourier's theorem, Laplace's principle of periodicity, and the projectile velocity which balances æthereal resistance at Sun's surface. The actions and reactions of centripetal gravitation and centrifugal radiation are thus coördinated in such ways as to give simple forms of expression for all kinetic correlations. Equations (4) and (5) represent similar tangential tendencies to belt formation by the *vis viva* of primitive tangential motion, both at the outer limits of the solar system and at the outer limits of the belt of greatest condensation. Equation (2) represents a harmonic relation of tangential velocities, at the chief nucleal centre and at the chief centre of condensation. This equation satisfies Laplace's demonstration of the tendency to exactness in simple numerical relations. It also satisfies various tendencies of subsidence as well as of linear and of rotary inertia. Equation (9) gives harmonic tangential velocities to the two interior companion masses, in the belt of greatest condensation. Equations (6), (7) and (8) represent radial and belt-rupturing tendencies of simple subsidence. In the mutual interactions of gravitating subsidence the sums of the gravitating accelerations, along mutually connecting lines, vary as the respective masses; therefore $\frac{mg^2}{2}$, or the *vis viva* of subsidence, varies as m^3. Equation (3) represents harmonic interactions between the centre of primitive subsidence (m_8) and the chief centre of condensation (m_3). The importance of these interactions is still further exemplified by the fact that $(t_\beta \div t_a)^2$ = Earth's oblateness according to Listing's estimate (Note 440). This accordance seems calculated to throw great doubt upon Delaunay's hypothesis of retardation by the "tidal brake."

462. *Deduced Values.*

The following harmonic values satisfy the equations of Note 460. Some of the latest astronomic estimates are also given, in order to show the closeness of accordance:

	Harmonic.	Astronomical.	
$m_0 \div m_1$	4527977	4512885	Encke.
$m_0 \div m_2$	387066	396256	Hill.
$m_0 \div m_3$	329196	329161	Faye.
$m_0 \div m_4$	2867780	2869157	Hall.
$m_0 \div m_5$	1049.4	1050	Leverrier.

	Harmonic.	Astronomical.	
$m_0 \div m_6$	3510.7	3512	Leverrier.
$m_0 \div m_7$	22508.7	22600 ± 100	Newcomb.
$m_0 \div m_8$	19384.6	19380 ± 70	Newcomb.

463. *Evidence of Nebular Subsidence.*

The outer portion of the Neptune-Uranus belt is harmonically connected with the belt of greatest condensation, as we have seen (Note 440), by an important mass-relation. One linkage of the inner portion of the same belt was given in Note 442; another is found in the proportion

$$t_\alpha : t_\beta :: (\rho_3 + \tfrac{1}{2}\rho_0)^2 : \rho_7^2.$$
$$1 : 366.2565 :: (1.00233155)^2 : (19.18244)^2$$

Leverrier's estimate of ρ_7 is 19.18264, which differs from the harmonic estimate by less than $\frac{1}{4}$ of Sun's semi-diameter. This harmony introduces: (1) The rupturing tendencies of nebular subsidence through $\frac{1}{2}r$; (2) The interstellar parabolic influences which have determined the harmonic positions of the eight primary planets and of the asteroidal belt (Note 46); (3) The conversion of parabolic into elliptical influence, with foci at the centres of Earth and Sun; (4) The variation of the times of nebular rotation inversely as the square of radius. These relations, taken in connection with equation δ, Note 460, furnish conclusive evidence in support of Herschel's "subsidence theory."

464. *Earth's Dependence on Luminous Undulation.*

The influence of luminous undulation in determining Earth's orbital period is quite as remarkable as its influence upon the time of solar rotation. The latter represents the maximum energy, while terrestrial revolution represents the mean energy of luminous undulation, in accordance with the general principle that, when a disturbance consists of terms involving sines or cosines of angles which vary with the time, the maximum energy is twice the mean energy. According to Stockwell, the secular centre of the belt of greatest condensation is at $1.0169394_{\rho 3}$, which is an arithmetical mean between Earth's mean radius vector and its mean aphelion. Earth, like Jupiter, shows the energy of æthereal projection as well as the mean energy of luminous undulation. We find, accordingly, $g_3 \times 1 \text{ year} = 1.0338788 v_\lambda$. This gives $v_\lambda = 185501.5$ miles.

465. *Jupiter's Dependence on Luminous Undulation.*

The combined influence of luminous undulation and central condensation, in determining Jupiter's orbital velocity, is equally striking.

$$1.0169394 r_3 : \rho_3 :: \sqrt{\gamma_5 \rho_5} : v_\lambda.$$

Substituting the harmonic value of ρ_3 (92542800), this proportion gives $v_\lambda = 185498.1$ miles.

466. *Neptune's Dependence on Luminous Undulation.*

The varied harmonies which have been pointed out between the Neptune-Uranus belt and the dense belt, may naturally lead us to seek for some additional evidence of luminous influence at the outer limit of the solar system. Such evidence is furnished by the proportion :

$$\rho_\alpha : \rho_\beta :: \rho_\gamma : \rho_3.$$

In this proportion ρ_α = Neptune's secular aphelion + Earth's mean aphelion ; ρ_β = Neptune's secular aphelion ; ρ_γ = centre of belt of greatest condensation. Substituting Stockwell's apsidal elements and the harmonic value of ρ_3, the proportion gives $v_\lambda = 185492$ miles. The extreme range of discrepancy in the three estimates (Notes 464–6) is less than $\frac{1}{1\,9\,4}$ of one per cent, or less than ten miles per second.

467. *Harmonic Estimate of the Constant of Aberration.*

Earth's mean orbital velocity is $1296000'' \div 31558149 = 0''.041067$; $v_\lambda = 185497.2$ miles, if we take the mean of the three foregoing estimates. To find the constant of aberration we have the proportion :

$$185497.2 : 92542800 : : 1 \text{ sec.} : 498.89 \text{ sec.}$$
$$0''.041067 \times 498.89 = 20''.488.$$

The close accordance of this value with Nyrén's estimate (Note 449), furnishes satisfactory confirmation both of the accuracy of Stockwell's calculations and of the precision of the harmonic estimate.

468. "*V.*"

The foregoing results abundantly show that the principle of harmonic motion is of "immense use not only in ordinary kinetics, but in the theories of sound, light, heat, etc." * Analogy, the law of parsimony, the theories of kinetic correlation, and the various principles enumerated in Note 461, all point to the value of v_λ, which is given in the foregoing note, as equivalent to the ratio "*V*" between the electrostatic and electromagnetic units of electricity, electric current, magnetic potential, electric displacement, surface density, magnetic force, and strength of current at a point. The corresponding resistance in the field of rotation is 29.853 Ohm's. This agrees very closely with Foucault's estimate, 29.836 Ohm's. Weber's estimate† was 31.074 ; Thomson's‡ 28.2 ; Maxwell's‡ 28.8 ; Michelson's§ 29.982. The modulus of light which is represented by the harmonic constant of aberration may be found as follows : $r_0 = \rho_3 \div 214.4513 = 431532.8$ miles ; $g_0 = m_0 r_3^2 \div m_3 r_0^2 = .16878$ miles ; $\pi r_0 \div v_\lambda = 7.3085$ sec. ; $7.3085 g_0 = 1.233$ miles $= v$ of solar rotation ; $\sqrt{g_0 r_0} = 269.82$ miles ; $269.82 \div 1.233 = 218.833$; $(218.833\pi)^2 r_0 = 472633 r_0 =$ modu-

* Thomson and Tait, *Nat. Phil.*, i, Sec. 52.
† *Pogg. Ann.*, Aug. 10, 1856.
‡ Rept. Brit. Assoc., 1869, pp. 434, 436.
§ *Am. J. Sci.*, Nov., 1879.

lus of light at Sun's surface; Laplace's limit, $L = 218.833^{\frac{2}{3}} r_0 = 36.314 r_0$. An additional linkage of Sun, Earth, Jupiter and Neptune is shown by the proportion:

$$L : \rho_5 :: \rho_{3.2} : \rho_{8.2}.$$

In this proportion $\rho_{3.2}$, $\rho_{8.2}$ represent secular perihelion of Earth and Neptune, respectively. Substituting Stockwell's value of $\rho_{3.2}$ gives $\rho_{8.2} = 29.68\rho_3$. Stockwell's estimate is $29.73\rho_8$.

469. *Another Linkage of Jupiter and Earth.*

Let $v_a \div$ *vis viva* of circular orbital revolution which Jupiter, when at mean aphelion, would be able to give to a particle at the centre of the solar system $= m_5^2 \div \rho_a$; $v_3 =$ molecular *vis viva* which Earth would be able to communicate to the same particle $= \frac{2}{5}(m_3^2 \div \rho_3)$; $v_\lambda =$ velocity of light; $v_3 =$ circular orbital velocity of a particle at the chief centre of condensation in the solar system (Earth). Then we find $v_a : v_3 :: v_\lambda : v_3$. Substituting the harmonic values of Earth's mass and of the constant of aberration, with Stockwell's estimate $\rho_a \div \rho_3 = 5.427351$, we find $v_\lambda \div v_3 = 10067.61$; $m_5 \div m_3 = 313.61$; $m_0 \div m_5 = 1049.69$, which differs by less than $\frac{1}{38}$ of one per cent from the value which was deduced in Note 462. This relation shows that, when nebular subsidence and luminous undulation had established incipient orbital motion around the Sun at Jupiter's mean aphelion, the actions and reactions among nucleal centres established the molecular motion, at the chief centre of density, which resulted in Earth's orbital oscillation. The first indication of the importance of the factor $\frac{2}{5}$ appears to have been given by me (Proc. Amer. Phil. Soc. xii, 394). Maxwell subsequently adopted it (P. Mag., June and Sept., 1877, pp. 453, 209) without leaving any record of the source from which he derived it, or of the reasons upon which it was based. In all of my investigations my first inquiry is, what obvious radial or tangential velocity, momentum, or *vis viva* is there, which would be likely to operate in producing or maintaining such exactness of cyclical harmony as is necessary for the stability of the system. The first trial usually gives some clue which suggests the next. These alternations, between Baconian observation and the "scientific use of the imagination," have been rewarded by frequent confirmations of predictions which I had hazarded and recorded weeks, months, or years, before they could be sustained by any known data.

470. *Dense-Belt Projection.*

The interstellar parabolic trajectories, which have tangential directices at Sun's surface, and a common focus at Sun's centre, have a vertical locus at $\frac{r_0}{2}$. The length of the luminous undulation which becomes semi-circular in solar rotation is πr_0. The sum of these two centrifugal

tendencies is $3.641593r_0$, or .0169414 $\left(\rho_3 + \frac{\rho_0}{2}\right)$, which differs by less than $\frac{1}{84}$ of one per cent from Stockwell's estimate of the projection of the centre of the dense belt. For the influence of $\rho_3 + \frac{\rho_0}{2}$ upon the orbital locus of Uranus, see Note 463.

471. *Sun's Equatorial Acceleration.*

The luminous projection of Jupiter and of the centre of the dense belt, as well as the other evidences of apsidal influence upon planetary harmonies, show that molecular *vis viva* has slightly modified the simple undulatory *vis viva* of the luminiferous æther. The amount of the solar equatorial acceleration is not precisely known, because no Sun-spots have been observed very near the solar equator; it cannot, however, differ much from Earth's mean secular eccentricity. We may, therefore, regard this as another evidence of the harmonic importance of "subsidence" to the belt of greatest density.

Stated Meeting, June 20, 1884.

Present, 5 members.

President, Mr. FRALEY, in the Chair.

A letter accepting membership was read from James H. Hutchinson, M.D., dated May 16, 1884, No. 133 South Twenty-second street, Philadelphia.

The decease of Prof. Dr. Heinrich Robert Göppert, at Breslau, May 18, 1884, aged 83, was announced.

A letter was received from J. M. Da Costa, M.D., accepting the appointment to prepare an obituary notice of the late Dr. Gross.

Prof. Baird requested, by letter, a copy of Proceedings No. 110 for the Library of the Imperial Museum at Strasburg, which was ordered to be sent.

Prof. E. D. Cope requested, by letter, the insertion of a paragraph in the Proceedings enlarging the notice of the minutes of January 18, 1884.